# Planning for Progression- The debate around the use of the elimination method in the teaching of simultaneous equations 

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#### Abstract

This research discusses the teaching of simultaneous equations with a focus on the merits of the elimination method and preferences for this method. This topic was taught in a sequence of six lessons to year 10 students. Three methods were taught: elimination method, substitution method and graph method. During these lessons I utilised the school iPads and the method of SOLO taxonomy. I analysed students' preference of method where the question allowed for choice, and their ability to answer the question where the method was specified. Students developed an understanding of the elimination method in order to use it effectively, developed a preference for this method and struggled with the substitution and graph methods.

This topic was taught in light of conflicting literature regarding the benefit of teaching the elimination method at GCSE. Some students struggled with rearranging formulae and complex substitution, leading me to conclude that the elimination method is the most appropriate for year 10 students. Given the impending changes in the GCSE exam, notably the inclusion of simultaneous equations in the foundation tier paper, I feel that the elimination method is a valuable method to teach to students of all levels and should be maintained in GCSE teaching to enable less able students to solve simultaneous equations and gain marks.


## Introduction

Algebra has long been seen as a boring, difficult aspect of the mathematics curriculum (Samo: 2009). Students develop misconceptions early (Samo: 2009) and this results in many adults claiming that this is the area of maths that they understand the least and which they use little in the real word (Murray: Year unknown). However, regardless of this apparent lack of understanding, students can get a good grade at GCSE without having mastered algebra (OFSTED: Made to Measure) and therefore many A-level students are 'underprepared' (BERA press release: 2014, page 1) for it at the post-16 level.

There is little mention of algebra in the Key Stage 1 curriculum (Department for Education: 2013), but students are introduced to the 'language of algebra' in Key Stage 2 (Department for Education: 2013, page 30) and it will continue to feature heavily in Key Stage 2 SATs (Standards and Testing Agency: 2014). These skills are built upon in Key Stages 3 and 4 (Department for Education: 2014). At the school where I will teach a sequence of lessons, 'basic algebra' is included in the year 7 , year 8 and year 9 schemes of work. The topic I have chosen to focus on, simultaneous equations, is introduced as the last topic in year 9 , continued in year 10 and then included in the year 11 higher scheme of work. I will be teaching this sequence of lessons to a year 10 , set 3 class. Therefore their expected baseline knowledge includes working with letters as numbers, solving equations and substitution. They should already know what simultaneous equations are and have some recollection of the methods used to solve them. It is easy to misjudge what students can remember and what their strengths and weaknesses are (Larcombe: 1985), therefore their baseline knowledge will be re-evaluated during the first lesson.

## Citation

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One reason that I have chosen to focus on simultaneous equations is that, unlike some algebra topics, it can be related to real life. This is in contrast to a concern about teaching maths, which is that it is removed from the real world (Kearney: 2013). Any topic that can be related to a real life context should be, in order to engage students, especially those who do not normally enjoy maths.

Samo (2009, page 31) found that students have many misconceptions, regarding the use of symbols in algebra, which impact on their learning. Making sense of these terms is fundamental in mathematics, and allowing students to relate them to real situations will be beneficial. The questions used to introduce the topic, do not have to use ' $x$ ' or ' $y$ '. They could use fish and chips, iPads and iPhones, games and games controls. Students could create variables themselves. This will help to negate the problem of not understanding symbols and letters.

Another reason that this topic is interesting is that three different methods for solving simultaneous equations can be taught: elimination, substitution and the graph method. This variety allows students to reflect on their learning, and for some questions, decide on a preferred method. Allowing students a choice of methods for some questions will encourage them to assess the ease and effectiveness of all three methods. This process of reflection is vital for students, enabling them to consolidate their knowledge and make connections between topics (Blackhouse et al: 1992). Students will be able to evaluate the methods in light of their own strengths and weaknesses and select the method that they can apply most effectively.

My analysis will assess:

- Their preference of method where the question allows for choice
- Their ability to answer questions where the method is specified

A lot of the questions covered during this sequence will be linear and this often allows for choice of method. However, when this topic is re-taught in year 11, some students will develop this knowledge in order to solve quadratics. This is where a preference for one method becomes problematic (Jagger: 2005). The problem originates specifically from students favouring the elimination method. This method cannot be used for quadratic equations. Therefore, if students cannot use the substitution method, they cannot progress on to harder questions, specifically ALevel (Jagger: 2005). It has been argued that, as this is the case, only substitution should be taught at GCSE (Jagger: 2005).However, I feel that elimination is still a valuable method for those students who will not progress onto quadratics as these students can still gain marks at GCSE. Most GCSE questions label the two equations (1) and (2), leading students to use this method (French: 2005). One Edexcel examiners report stated that the 'alternative' method of substitution was rarely seen (Edexcel: 2013, page 8). Therefore, elimination will still be taught.

By the end of the first four lessons, I aim for all students to be able to:

- Answer questions on linear equations using their preferred method
- Answer questions using a specified method

The majority of the class are predicted A's at GCSE with some A*'s and B's. There has been a lot of focus placed on this group recently as, during their most recent assessment, it was illustrated that they are working on par with set 5 . It has been identified that some students are making all of the correct notes without understanding the work. There is a growing recognition that students should understand maths and not just learn algorithms (Blackhouse et al: 1992). I feel that one of the most
important tools that can be used to facilitate this level of understanding is questioning. Questioning can be used to stimulate students to think as well as to test students' knowledge (Blackhouse et al: 1992). Therefore I will pay close attention to the questions I ask and the questions students ask in turn.

There will be six lessons in the sequence:

1. Simple introduction to the topic, using the elimination method with positive numbers. This is advocated by French (2005). This lesson can be used to assess what the students remember from year 9 and whether a preference for any method has already developed.
2. Harder questions involving negatives, still using the elimination method.
3. Introduction to the substitution method.
4. Introduction to the graph method.
5. A lesson based on SOLO taxonomy.
6. Assessment. Questions will test student's knowledge of all three methods.

By the end of the first lesson, students should have progressed to the stage where they recognise simultaneous equations questions and can solve them using the elimination method. This should be developed in lesson three to include the substitution method. They should progress to the stage where they can substitute equations in and then solve the simple equations that result. Due to the order of the scheme of work, students will not progress on to quadratics. The fourth lesson will develop a link between linear graphs and simultaneous equations. The progress made in these four lessons will be assessed in lessons 5 and 6.

The fourth lesson will allow me to make full use of the school's iPads. In an attempt to go 'paperless' all teachers are provided with iPads and students are encouraged to buy one. From my observations it appears that these are not utilised effectively. Attard (2013, Page 1) found that teachers have 'little to guide them' in terms of how to use iPads appropriately and that it is difficult to monitor whether students are on task or experiencing difficulties. It has also been noted that in lessons where iPads were used as an ineffective learning tool, there was no discussion of what would be covered, and then what had been covered (Attard: 2013). I will therefore include those key points in my lessons, to ensure that students understand the focus of the lesson, understand why the iPads are being used and understand what they need to achieve by the end of the lesson. This will maximise the effectiveness of the iPad as a learning aid. Despite these concerns, I feel that iPads can have a positive effect, especially when the lesson involves graphs. In order to solve simultaneous equations graphically the diagram needs to be extremely accurate, this can be very time consuming if hand drawn. Rossing et al (2012) found that one of the main benefits of iPads is their convenience and ease of use. Using an app on the iPad will allow students to answer a greater number of questions during the lesson. Using the iPads also allows for the lesson to become 'student centred' (Attard: 2013, page 3) and be easily catered to each student. They can move at their own pace, create their own equations, adjust the difficulty of the equations and have more time to investigate. Rossing et al reported that 117 students, out of 206, felt that the iPad suited their 'specific learning style, preference and speeds' (Page 12).

The fifth lesson will be designed using the SOLO taxonomy. This stands for 'Structure of Observed Learning Outcomes' and was developed to take into account factors such as students' prior knowledge and misconceptions (Potter: 2012, page 9). As Boulton-Lewis (1994) found, based on statements collected from students and lecturers, SOLO taxonomy provides a useful model for deriving different levels of knowledge and learning. Progress can be measured by which sections students complete and how successful they are.

There are five stages, of which four will be used during the lesson. The style of questions for each section is in brackets.

- Uni-structural (Any method can be used)
- Multi-structural (A specified method has to be used)
- Relational (Students have to identify the most appropriate method)
- Extended abstract (Students have to create their own equations from worded questions)

A lesson based on the SOLO taxonomy allows, and supports, students to reflect on their own learning (Trignano: 2014). They can assess their understanding of the different questions and identify any strengths or weaknesses. This will also aid students in identifying and reflecting on what the next steps of learning are.

Another benefit is in allowing teachers to thoughtfully shape learning outcomes (Trignano: 2014). The SOLO taxonomy clearly illustrates levels of understanding. If teachers use this and cover all stages, students will have their knowledge built on naturally. During the lesson students will have to move around the room to access the different questions, the teacher will instantly be able to see who is moving, who is not and which questions are troubling students. Teachers are therefore provided with useful qualitative and quantitative feedback immediately (Potter: 2012, page 9).

## Evaluation

## Outline

The next section will evaluate my progression map and the sequence of lessons. I will first discuss students' baseline knowledge and the sequence of lessons. I will evaluate them using the assessment results as evidence. I will then discuss:

1. Students' preference of the elimination method
2. Students' ability to answer questions where a method is prescribed

## Baseline

Prior to teaching this topic, students had lessons on substitution, rearranging formula and solving equations. This allowed me to assess their baseline knowledge. It became apparent during the substitution lesson that all students could substitute a single figure into simple equations and equations involving quadratics. However, the lesson did not explore students' knowledge of more difficult substitution, such as the kind expected of them in lesson 3. In hindsight, this lesson should have been used to extend their knowledge, as only one lesson was going to be spent on the substitution method. This would have provided the opportunity for students to practice this type of substitution without the extension of solving.

The lesson on rearranging formula provided good insight into how well students would be able to rearrange during the substitution and graph lessons in order to get equations in the form $y=$ or $x=$. During the lesson this was heavily linked to solving equations and students appeared to understand it relatively well. However, from marking the homework that was set (to consolidate this knowledge) it became apparent that students had not developed as deep an understanding as I first thought. This prompted me to use equations that needed rearranging as extension questions only.

It was originally intended that students would be introduced to quadratics during the third lesson. However, upon discussion with the host teacher, it was discovered that the quadratics topic had not yet been taught. Therefore students could not progress onto the type of questions that Jagger (2005) feels makes the substitution method most relevant.

Students' inability to answer quadratics or confidently re-arrange equations limited the progress they could make during this sequence of lessons. Future plans to rectify this barrier, and those discovered during the lessons will be discussed in a later section.

## Lessons 1 and 2

The elimination method was taught over two consecutive lessons. The lesson objective from the first lesson (see Table 1) was met by all students and the lesson objective from the second lesson (See Table 1) was met by the majority. Progress during this second lesson was hindered by students' lack of understanding of when to add or when to subtract equations. They also struggled with adding and subtracting negative numbers. A common mistake occurred when one of the equations involved a negative. For instance on question 6:

- $7 x+4 y=-68$
- $3 x-2 y=-18$

Students were able to multiply the equations correctly in order to make the y coefficients the same:

- $7 x+4 y=-68$
- $6 x-4 y=-36$

However, many students subtracted these two equations to get:

- $x=-32$

Rather than adding to get:

- $13 x=-104$

This illustrates an understanding of the idea that one of the coefficients has to be eliminated but a lack of understanding of how to do this. As a result, only $6 / 29$ students got full marks for this question, compared to $26 / 29$ that got question 2 correct, an elimination question involving two positive symbols. These results match general GCSE trends, with one Edexcel examiner's report stating that students who had experienced solving simultaneous equations were generally able to show evidence of using a correct method, although it was often 'spoilt' by arithmetic errors (Edexcel: 2012, Page 9).

## Lesson 3

The third lesson was spent covering the substitution method. By the end of the lesson, many students had achieved the learning objective, (see table 1) although progress was slower than in the previous two lessons. The starter was designed to recap simple substitution and assess how well they could adapt their knowledge in order to do more complex substitution:

$$
\begin{gathered}
x=y+5 \\
x+3 y=29
\end{gathered}
$$

It was apparent during this activity that some students would struggle with this method. The main barrier to learning was that students could not view $x=y+5$ as a value that could be substituted into another equation.

The students who did understand this were able to make the link between the new equation they had and the purpose of the lesson, in part due to the work done during lessons one and two. However, the lack of certainty shown in the lesson is evidenced in the assessment results. 12/29 students did not get full marks on the substitution question and $4 / 29$ students did not attempt it at all.

## Lesson 4

The fourth lesson introduced students to the idea of solving simultaneous equations by plotting graphs. The lesson objective can be found in Table 1.
After discussion with my mentor about the most effective way to use iPads for this lesson, it was suggested that part of the lesson be spent practising plotting the graphs of simultaneous equations. This had not originally been planned, as this can be time consuming and detract from the real aim of the lesson. This proved to be true. Students had to hand-draw the graphs for the first two questions, before moving on to using just the iPads to solve the equations. As a result, students did not progress as expected, as they were able to answer 2-4 questions rather than the 10+ they could have done if the lesson was solely iPad based.


The assessment results illustrate how this has affected student progress.
On question 8 (table 2) students first had to fill in a table of values, plot the line and finally solve the simultaneous equations.

No doubt the success of parts $A$ and $B$ were due to the structure of the fourth lesson. During the lesson, students could not remember how to fill in a table of values until advice was given. Yet on the assessment all students got full marks on a similar question. Part C is the most interesting. Although all students had filled in the table of values correctly and all but two had plotted the correct line, only 14 students, less than half, achieved the full two marks for part C despite merely having to read off the values for the point of intersection. Some students tried to solve the equations using the elimination method; some created new tables of values and some did not attempt it at all.

Students do not appear to appreciate that the point of intersection is a set of values that are true for both equations and are therefore the simultaneous solutions. This may be due to a lack of understanding for what a graph actually represents. The OCR examiner's report for June 2013 and 2014 found similar results, that tables and graphs were usually correct but that, when it came to using the graphs, students had less understanding and did not appreciate the significance of the point of intersection.

## Lesson 5

The fifth lesson was based on SOLO Taxonomy and was designed to assess the success of the aims set out in the introduction. The lesson objective can be found in Table 1.

The uni-structural section allowed students to use any method they liked to solve equations. No help was needed by any students on this section and when their books were marked, all answers were correct. It is therefore argued that the success of this section illustrates that the students have a good understanding of all three methods at a basic level.

The multi-structural section stated the method that had to be used for each question. The questions were more difficult than those on the uni-structural level. However, evidence from this section illustrates a good knowledge of all three methods. The only help needed related to arithmetic, not applying the specific method.

The relational section required students only to identify which method would be most appropriate to use and not to answer the questions. Although many students misread the instructions and attempted to answer the questions, from their answers it is apparent that students appreciate which method suits particular questions.

The extended abstract section required students to create their own simultaneous equations based on information presented to them. This was the most difficult section and students had not yet encountered questions of this type in the preceding lessons. The students who answered these questions were extremely successful. They were able to extract the relevant information and identify relationships in order to create solvable equations, which illustrates a deep understanding of the topic.

I do not feel that the SOLO lessons advanced my understanding of, or gave clear evidence of, students' knowledge or progress. The evidence gained from this lesson, that students understand all three methods for example, is contradicted by the assessment results and observations in lesson. Frankland (2007) argues that for the SOLO taxonomy to be effective, teaching activities, assessment tasks and criteria should be designed with reference to the taxonomy. Therefore, in hindsight, the entire sequence of lessons would have benefitted from being framed with SOLO in mind, in order for the fifth lesson to be successful. As a stand-alone lesson it did not work.

## If the topic was to be taught again

This topic was taught in light of contrasting pieces of literature regarding the benefit of teaching the elimination method at GCSE (Jagger: 2005, French: 2005). I still firmly believe that there are advantages of teaching this method. Jagger (2005) correctly states that the substitution method is where this topic naturally progresses and that it is vital for A-Level maths. However, experience from this group suggests that at the age of 14-15, when most students are just starting the GCSE curriculum, some struggle to grasp even the concept of more complex substitution. As I found, some students cannot relate $x=3$ and $x=y+5$ and therefore cannot make the connection between the second figure and substitution. This has significant ramifications for the progress students can make in year 10, and will be especially important to consider when teaching starts for the new GCSE in September 2015. This will be 'more demanding' (Edexcel: 2014) and the topic of 'solve linear/linear simultaneous equations' will be included in the foundation tier exam. This will require teachers to assess the most appropriate method for all students, not just those in the higher sets. I feel that this method is the elimination method.

If the sequence was to be taught again, only the elimination method would be taught in year 10 and the other methods would be taught in year 11. The host teacher has echoed this conclusion. This
would be preferable for a number of reasons. Firstly, students in year 11 are more prepared to challenge themselves with grade ' $A$ ' work and have covered more of the GCSE curriculum. This last point leads on to another reason for waiting. This group had not yet covered the quadratics topic; it was featured later in the scheme of work. Therefore, even if the students had understood the substitution method more quickly and to a higher level, it still would have been difficult to stretch them further, because they did not have the required baseline knowledge of quadratics.

During the elimination lessons, more time would be spent discussing the distinction between equations where the symbols are the same and those where they are different. The OCR examiners report for 2014 (page 10) found that even on a 'comparatively straightforward' simultaneous equations question, marks were lost because students incorrectly added to eliminate a variable. Using a simple number line, students would be shown clearly why equations are sometimes added and sometimes subtracted

Although understanding how to plot graphs is important, I feel that the fourth lesson could have benefitted from just using iPads to solve more equations. This would have changed the focus of the lesson from plotting graphs, a separate topic, to solving simultaneous equations graphically. This would have allowed students to understand that the aim of the lesson was finding the two values. Many of the students were focused on drawing accurate graphs and the significance of the point of intersection was lost, affecting the progress students made. A handful of students didn't use iPads at all.

## Preference of method

Students' preference was undoubtedly for the elimination method. For all questions on the assessment where a graph wasn't provided or the substitution method didn't have to be used, the elimination method was used. During the lesson 5 starter, although the substitution method was meant to be used for question 2 and the graph method was supposed to be used for questions 3 , those who were struggling turned to the elimination method. More evidence comes from the final question of the assessment. Students had not been taught how to solve quadratic equations and yet, with no instruction as to which method to use, all but two of the students who attempted this question tried to use the elimination method.

## Ability to answer questions where the method is prescribed

After analysis, I do not feel that students can confidently answer questions where the substitution or graph method has to be used. During the starter for lesson 5, many students resorted to using the elimination method where the substitution method and graph method were prescribed. On the assessment, question 2 stated that the elimination method must be used and only three students dropped marks here. However, to get marks on question 4, the substitution method had to be used. Ten students did not get full marks for this and four did not attempt it. While this evidence illustrates proficiency when the elimination method is prescribed, it also illustrates a lack of confidence in the substitution method. The only evidence in support of students having this ability is their success during the SOLO lesson. However, students were able to work in groups for this activity and therefore it cannot be stated with confidence that this illustrates that all students can answer questions where a specific method has to be used.

## Conclusion

This assignment aimed to discuss the teaching of simultaneous equations, with a particular focus on the merits of the elimination method and students preference of method. It has been found that, in year 10, students can develop an understanding of the elimination method in order to use it effectively, develop a preference for this method and struggle with the substitution and graph
methods. This may not be true of higher sets. My findings suggest, therefore, the teaching of the elimination method should be maintained in GCSE teaching. This opinion is in line with that of French (2005) who felt that the elimination method was not only a useful way of introducing the topics, but also provided more students with the opportunity to make progress and gain marks at GCSE. It may be the case that individual schools and departments need to standardise the teaching of this topic, especially in preparation for the more rigorous GCSE, which will demand less able students to be able to solve simultaneous equations. However, I feel that the elimination method best prepares the majority of students to gain the most possible marks.

One suggestion going forward may be to introduce 'method consistency' among classes within a school, meaning that certain sets learn substitution and elimination and some just learn elimination. This will allow students to be taught the same one or two methods in year 9 , year 10 and year 11, rather than learning different methods depending on the teacher. Ofsted's 2012 report 'Made to Measure' (page 55) rarely saw agreed approaches to teaching topics or sequences of related topics but felt that this was a key starting point in improving the quality of weaker teaching and consequently the opportunity for all students to make good progress. Despite this research being conducted in the UK, maths is a universal language. Therefore the lessons learnt here are applicable to the worldwide teaching of maths.

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