'Noticing' examples presented in primary mathematics textbooks

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Clare Tope
Open University

Abstract
There is broad agreement that the choice of mathematical examples is central to the teaching of mathematics and that a teacher's choice of examples may either facilitate or impede learners' understanding. From this starting point it can be deduced that programmes of instruction for teachers should include the study of mathematical examples. However, there is much to learn about the kind of professional development that might support beginner teachers in the careful choice of examples. This paper presents a case study of a group of sixteen students on a one-year Primary PGCE (5-11) programme who met each month over the first six months of their course to consider example sets. The research events explored how student responses to examples presented in textbooks changed over time. The outcomes of this study suggest that over time students noticed more about opportunities for generalisation and progression in the mathematical constructs explored in the example sets. Students also noted many of the pedagogical concerns associated with textbook use. The outcomes of this study lead to the recommendation that programmes of instruction include explicit opportunities for beginner teachers to generate, explore and reflect upon examples for real or hypothetical classroom situations.

Key Words
Primary; mathematics; notice; examples; textbooks.

Introduction

Watson and Mason (2005:5) provide a line of argument in support of the possible contribution of teacher choice of examples to successful lessons, with their suggestion that the careful choice of examples 'might be seen as central in the learning and teaching of mathematics'. This was echoed in two school visits which both highlighted the need to provide further opportunities to support students in their choice of examples. During the first lesson a beginner teacher presented a group of children with a sequence of very carefully planned examples with the potential to expose and explore aspects of place value. In the discussion which followed the lesson, the student was able to indicate an intention to introduce variety into the questions posed but was unable to articulate a more detailed rationale for the decisions she made. This was followed a few weeks later by a visit to a student who was experiencing difficulty. When evaluating the lesson, the student commented, 'my choice of examples was sloppy'. This indicates that in the student's own judgement part of the reason for the lack of success of the lesson was his choice of examples. In summary, there is both anecdotal and research evidence to suggest that programmes of instruction for teachers should include the study of examples.

This paper aims to contribute to the discussion about effective teacher education in the careful choice of examples. The first steps in developing the research study were to identify an approach to professional learning and a source of mathematical examples. Potential solutions to these two dilemmas were found in the work of Mason (2002) and Ma (1999).

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Mason (2002:144) advocates 'noticing' as a way of prompting and sustaining professional learning. His argument is based on the premise that effective change (or learning) is something that people do for themselves when they adopt an enquiring stance towards their experience. He goes on to suggest that effective change depends on individuals becoming aware of (noticing) possibilities that were not previously available. He also argues that this kind of change can be supported by the exchange of accounts with a group of people whose presence provides a sounding board for conjectures and theories. He uses the term 'validating with others' to summarise this process (Mason, 2002:95).

Ma (1999) suggests that the study of textbooks forms a significant part of programmes of instruction for teachers in China. She notes that teachers examine the choice of examples in a unit, why these examples were selected and why the examples were presented in a particular order. She argues that through this process, their knowledge of both what to teach and how to teach grows. This is echoed in the outcomes of a more recent extensive transnational study by Cambridge Assessment (2016) which concluded that high-quality textbooks have the potential to play a key role in mathematics education. Thus, the proposal for the project was to explore sets of examples presented in textbooks, guided by 'The Discipline of Noticing' (Mason, 2002:3).

From this broad starting point, three research questions emerged:

- What do beginner teachers notice about sets of examples presented in textbooks?
- How does this change over time?
- What are the implications for mathematics teacher educators?

Literature Review

Variation

Von Glasersfeld (1985) argues that knowledge is not passively received by the learner, but is actively constructed through a combination of experiences and cognition. From this starting point it can be deduced that learners' new understandings are dependent on prior knowledge and experiences i.e. what each person knows is the accumulation of all their experiences so far and each new encounter either adds to that understanding or challenges it. Marton and Booth (1997) develop this line of argument and propose that there needs to be variation in what learners' experience in order for meaning to be constructed. Evidence in support of this claim can be found in the work of a range of authors. For example, Skemp (1971) argued that the criterion for understanding a concept is that of behaving in a way indicative of classifying new data according to the similarities which go to form this concept. More recently, Leung (2003) points out that discernment comes about when the key features of the concept being focused on are temporarily demarcated from the whole. This is characteristic of what Gu (1981) calls 'teaching with variation' and more specifically 'procedural variation'. Procedural variation is a particular type of repetitive learning which aims to extend conceptual understanding by varying one condition in the original problem and then reflecting on the implications of that change.

Examples and Generalisation

Watson and Mason (2006:3) offer the following definition of an example as it might be used in mathematics education: 'an example is anything from which a learner might generalise'. Thus, examples are a particular case of a larger class, from which one can reason and generalise. Clearly, no single example is sufficient to provide learners with the full extent of what is possible and may in fact be misleading. For example, a single example of a right angle presented as the meeting of a horizontal and a vertical line obscures the generalisation that a right angle is a measure of turn of ninety degrees. This indicates the need for collections of examples that manifest the full scope of mathematical ideas and allow for generalisations to be tested. The boundaries for the sets of examples lie in the dimensions of possible variation and the associated permissible change in each of those dimensions (Doerr, 2000). For example, the number of different triangles is limited to three (equilateral, isosceles and scalene) because a triangle is a three- sided polygon and is bounded by the dimensions of possible variation in the number of equal angles (and sides). The range of permissible change in the size of the angles is bounded by the requirement for the three angles in a triangle to add up to one hundred and eighty degrees. Generalisations can be tested against these boundaries by drawing on different kinds of examples.

Polya (1981:10) makes use of the terms 'extreme' and 'representative' examples to explain this process. A reference example is one that becomes extremely familiar and can provide a starting point for conjecture. Using 'extreme' examples means going to the edge of a set of boundaries and seeing if anything unusual happens or if the conjecture holds true. Zodik and Zaslavsky (2008) draw attention to the use of both non-examples and counter examples as a means to test generalisations and deepen understanding of mathematical ideas. Askew and Wiliam (1995) use triangles to illustrate the role of non-examples and suggest that in order to discern what is a triangle children need to be asked to 'rule in' and 'rule out' a range of 2-d and 3-d shapes, in order to identify the key features of a triangle.

Pedagogy of textbook use

The line of argument here is that there is an overlap between examples presented to children in textbooks and more general pedagogical concerns. This might include: decisions about the range and extent of change in a single set of examples or in a sequence of pages in a text book; knowledge of previous foci in the text book series; whether the exercise is intended as revision or exploration and when is it appropriate to make generalisations implicit and when is it necessary to be explicit (Mason, 2017:408). This leads to a proposal that appropriate teacher intervention is usually required in order for individual meaning making to occur. This indicates the need for discussion which invites learners to reflect on the outcomes of their actions, so they recognise relationships and use these to make generalisations (Simon and Tzur, 2004). This idea is developed by Cambridge Assessment (2016:16) who highlight the importance of deliberate recording of 'what has been learned' as one of the key principles for designing high-quality textbooks.

Research Approach

Institutional and programme approval was gained to undertake the study.

The best-fit description of the research approach is 'an instrumental case study', as the purpose of the case was to allow the pursuit an area of interest (Denzin and Lincoln, 2011:437). Step one in the research process was, therefore, to identify a group of participants or a 'case' to study (Stake, 2000:435). The full cohort of students on a one-year Primary PGCE (5-11) Programme were offered the opportunity to be part of the research group. All students who gave their permission to be involved were invited to join the research group. A research sample of fifteen students with varied mathematical backgrounds was secured. For the purposes of undertaking and reporting on the outcomes of this study these students are considered as a focus group (rather that fifteen individuals).

Interim findings from the Textbook Project undertaken by NCETM (2015), report positively on the outcomes for students using the 'The Shanghai Maths Project Book' (Fan, 2016). This text was adopted as the core source of extracts for student discussion. Typically, an enquiry draws on a process in which key variables are changed systematically at each consecutive research event. However, in this case, it was not possible to predict what the students might notice in each example set. Also, the intention was to examine the changes in student responses as they emerged from their commentary. For these reasons the choice of the order of example sets focused on areas of mathematics in which students' confidence was likely to be high in the first instance and proceeded with more complex areas of mathematics.

The project comprised of six research events over a six-month period which each took the same format. Each research event was planned to allow students to engage in the discipline of noticing; in particular, recognising possibilities and then seeking resonance with others (Mason, 2002:94). Students were presented with a set of examples from a textbook and asked to provide an individual written response to the question ‘what do you notice’? Students then collaborated with another member of the group to exchange ideas and add further annotations to their written responses. The third step in each research event was an open-ended focus group discussion in which students considered the same question.

Analysis of the data sought to identify and report patterns or themes that emerged from responses to the tasks (Braun and Clarke, 2006). The tool selected for this task was ‘descriptive coding’ in which codes were derived from the language of the text (Savin-Baden and Major, 2013). An iterative process of coding was employed. The first step was to number and list written responses from several students verbatim. The original labels were then slowly refined by reading and re-reading and comparing with other student responses until patterns could be identified and clustered in labelled categories. Students were labelled alphabetically, and the outcomes of the coding activity were recorded on a series of spreadsheets which also allowed for quantitative analysis. However, the ‘keyness’ of a theme was not dependent on quantifiable measures alone but rather on whether it captured something important in relation to the overall research questions (Braun and Clarke, 2006). The audio recordings were used to verify (or otherwise) the data analysed from the written responses and therefore triangulate the findings. Analysis of the recordings included the identification of brief excepts which added further insight into issues that were highlighted by the coding activity; some of these extracts are included in the analysis below.

Data Analysis

Analysis of the research events revealed that collectively, over time, students noticed three things which are explored in the next three sections:

1. Opportunities for mathematical generalisation;
2. Progression in questions posed;
3. Pedagogy of textbook use.

Opportunities for mathematical generalisation

In order to illustrate a developing awareness of the potential for mathematical generalisations to be made, student responses to three consecutive research events are considered. Figure 1. sets out the three example sets that students were asked to explore in these research events.

2 Fill in the table.

Addend	1327	3204	584	1178	1257	9178
Addend	1150	2328	4265	7433	4465	822
Sum						

2 Fill in the ◯ with >, < or =.

$87 \div 3$ ◯ $78 \div 3$ $150 \div 5$ ◯ $105 \div 5$ $264 \div 4$ ◯ $272 \div 4$

$75 \div 5$ ◯ $75 \div 3$ $392 \div 2$ ◯ $392 \div 8$ $756 \div 8$ ◯ $756 \div 7$

$96 \div 8$ ◯ $84 \div 7$ $650 \div 5$ ◯ $990 \div 9$ $610 \div 5$ ◯ $738 \div 6$

2 Compare the fractions and write >, < or = in the brackets.

(a) $\frac{1}{63}$ ( ) $\frac{1}{36}$ (b) $\frac{3}{20}$ ( ) $\frac{3}{80}$

(c) $\frac{8}{100}$ ( ) $\frac{8}{99}$ (d) $\frac{7}{41}$ ( ) $\frac{7}{40}$

(e) $\frac{25}{200}$ ( ) $\frac{25}{300}$ (f) $\frac{11}{535}$ ( ) $\frac{11}{553}$

(g) $\frac{60}{601}$ ( ) $\frac{60}{699}$ ( ) $\frac{60}{700}$

Figure 1. Example Sets (Fan,2016).

The first set of examples included an opportunity for students to notice that one of the examples did not require 'exchange' across any of the columns. Analysis of both the written and verbal commentary suggests that students did not notice (or label) this particular example or consider more broadly the potential value of this example in supporting a generalisation about when the need for exchange could be 'ruled in' or ruled out' (Askew and Wiliam,1995). Instead, at this early stage of the research process, the two most commonly coded categories were 'page layout' and 'pupil difficulties'.

In the second example some students recognised that there was an invariant feature in rows one and two and drew this to the attention of the group. This suggests that, while student understanding of systematic variation was not complete, there was a development from the previous week and the group did notice that features of the question had been fixed (Gu,1981). However, there was no evidence the students were aware that this provided an opportunity for generalisations to be made.

In the third set of examples students were able to identify and label opportunities for generalisation. As Extract 1 below indicates, one student noticed the invariant feature in the numerators. She noted in her written commentary that 'these examples have fixed one variable'. Another student introduced a debate about the need for a 'reference' example and the discussion continued to include the potential for a generalisation to be made.

Student L	The numerator was always the same, but not always one.	The invariant feature of the examples is noted
Student B	But then there was nothing smaller, like a ¼ or 1/5 which they might have been more used to looking at in school	Questions the need for a 'reference' example against which future examples can be tested
Student C	I think that in that respect, in this case, it is better to have the big numbers so that they are not picturing them	
Student L	But then when they do get it, they realise that it is universal, and they will know that no matter how big the number gets on the bottom they will always be able to tell which fraction is larger	The term 'universal' is introduced as way to describe generalisation
Student F	If you wanted to, the children could actually make the generalisation , then you need some of those bigger numbers because they need to know that it works with unfamiliar fractions; so if it is fractions that they are used to then they may not know what to do when they get something like 61/ 700 or 11/553.	Awareness of the need for the generalisation to true in all cases rather than specific to familiar fractions

Figure 2. Extract 1: Focus group discussion.

The discussion then developed into a debate about the conditions that might be required in a set of examples in order to facilitate children making generalisations. This leads to the tentative conclusion that students had noticed that in order to extend conceptual understanding there is a need for a change or variation in order to test generalisations (Gu, 1981).

Student E	One issue I had with these questions is that people have said 'and then the penny dropped' and once that had happened then you did not need to do any more maths, you just had to identify which denominator had the bigger value and draw the symbol, so it was useful for making generalisations but there is not a lot of thinking going on	Debate the contribution of automisation to generalisation
Student F	Ah, but you could argue it the other way that making generalisations is the thinking and then testing it for different kinds of numbers	Two-way relationship between procedural and conceptual understanding
Student E	7 or 8 examples of the same thing is too much it becomes a trick	Conditions to facilitate generalisation
Student M	I think you need to make sure that you have enough to grasp the concept because if you don't have enough then you don't get the idea and like you said it becomes a trick but if you have enough then you change something it works	Need for variation to extend conceptual understanding

Figure 3. Extract 2: Focus group discussion.

The discussion concludes with a recommendation of an amendment to the textbook to provide an opportunity for learners to articulate a generalisation about the relative size of fractions with the same numerator. Here the content of the conversation indicates that students have noticed the need for learners to reflect on the outcomes of their actions which Cambridge Assessment (2016) propose is an important and often neglected feature of mathematics textbooks.

Student B	It would have been good if we had.. you remember when we had the sentences to fill in … so you could have the same thing here where children have to write down what the generalisation is
Student G	I think my objections would be satisfied if there was a sentence you had to write about what you did each time and why

Figure 4. Extract 3: Focus group discussion.

In summary, over the course of the three research events students developed an increased awareness of possibilities for generalisation. In broader terms, relating to the discipline of noticing, students moved to a position where they were able to identify and examine key ideas within a complex situation.

Progression

In order to illustrate developments in students' observations about progression this section compares what students noticed in the first and last research events.

In the first research event students were asked 'what do you notice' about the set of examples in Figure 5. below which is annotated to indicate what was available for students to notice.

Addend	1327	3204	584	1178	1257	9178
Addend	1150	2328	4265	7433	4465	822
Sum	No exchange required	Exchange from units to tens	Exchange from tens to hundreds	Exchange from units to tens and tens to hundreds	Exchange from units to tens and tens to hundreds	Exchange from units to tens, tens to hundreds to thousands and thousands to ten thousands

Figure 5. Annotations to indicate what was available for students to notice about progression.

One of the more startling outcomes of Research Event 1 was that analysis of individual written comments suggested that a significant number of students did not recognise progression in the examples. Askew (2004) acknowledges that the extent to which examples are transparent is subjective and suggests that pupils will always interpret examples in the light of previous experiences. The evidence would seem to suggest that the same applies to some beginner teachers. Previous experiences may have led this group of beginner teachers to identify progression with an increasing number of digits in the question posed (rather than the requirement for exchange) and so changes in the complexity of the examples were not immediately apparent.

Even at this early stage in the research process the value of group discussions became evident and by the end of the first research event the group came to realise that there was a carefully planned progression in the examples.

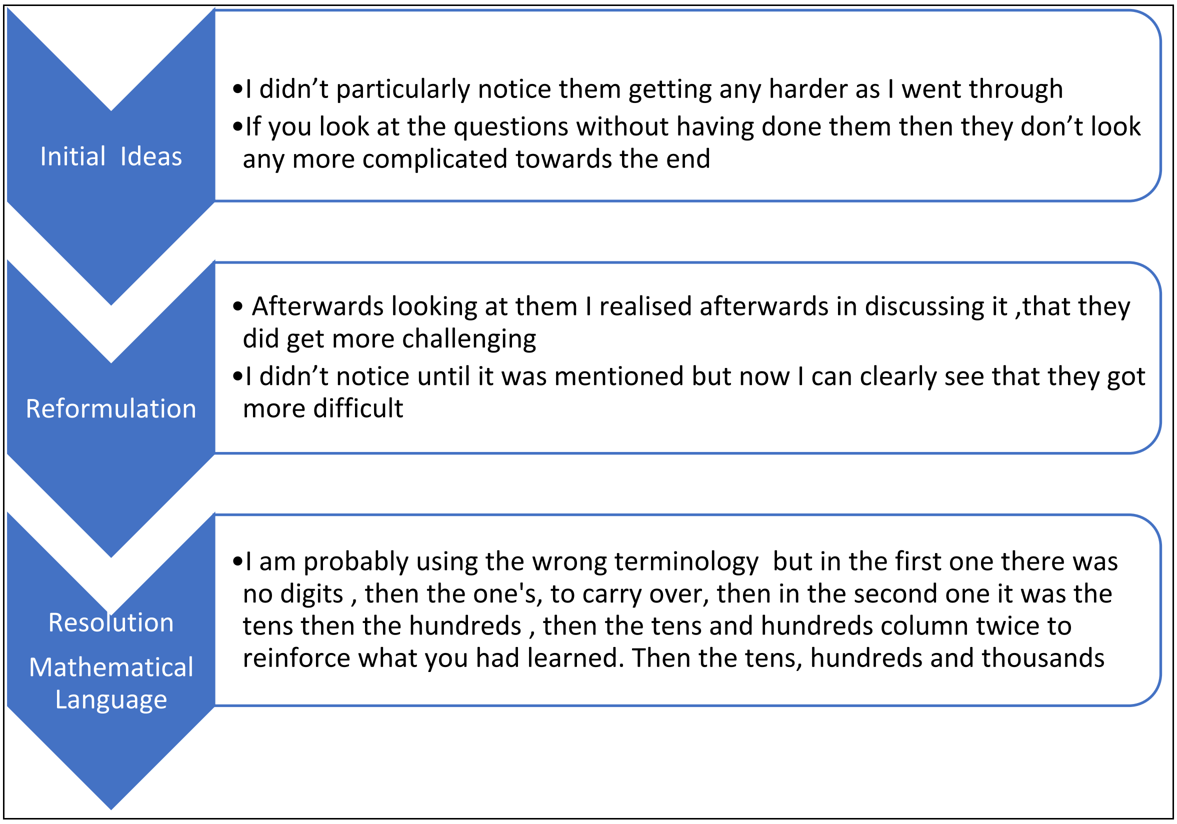

Figure 6. Focus group identification and confirmation of progression.

In the final research event, student’s discussion of progression developed to include detailed commentary on aspects of progression. Extract 4 below includes commentary about the changes from one set of questions to the next on a page in the textbook.

Student H	We noticed that is jumps straight from hundreds to ten thousands and didn’t include the thousands
Student B	I thought that the progression was too rapid and that you could have one page on the hundreds alone and then move on
Student I	There is very clear scaffolding happening because you have got your number line with your number line on and your hundreds and then you have got your number line with 10 thousand , And then the next step is that all of that is gone completely; the grid is gone, and then you have got just the numbers with the stuff in the middle.
Student J	and It wasn’t just the numbers that were changing it was the type of question. It might be better if it stayed in the same sort of format for each question if you were changing the numbers, so you are confusing two things rather than just one.

Figure 7. Extract 4.

The outcomes of this analysis lead to the tentative suggestion that student commentary about progression deepens to include possible anomalies in the progression steps. It also broadens to include analysis of the number of examples that are needed in order for meaning making to occur at each step in the progression. Students also noticed that the inclusion and then removal of visual images also represented sequential change in the level of difficulty in the task.

Pedagogy of textbook use

A number of other things that students noticed can be clustered under the heading 'pedagogy of textbook use'. Over time student commentary developed to include discussion about where the examples fitted within a unit of work, as one student stated, 'we don't know if this is the first time someone has approached this or if it is a consolidation task or if somebody has already got the prior knowledge'. This suggests that over time, another dimension of noticing is added; students recognise the importance of context in learner responses to an example set. Students also noticed aspects of the layout or instructions that had the potential to either promote or hinder understanding. For example, several students completed the division questions in Figure 1 in columns and did not realise the potential to make any generalisations about the outcomes of division. In another example set, one student commented 'you just do one and move on'. This led to the realisation that there is potential for the layout to encourage comparison, conjecture and generalisation between sequences rather than merely completing questions separately (Watson and Thompson, 2015). Finally, students noticed when the language used in the textbook lacked the precision that Cambridge Assessment (2016) suggested was an important feature of textbooks. For example, one question asked, 'to which whole hundred number on the number line is each of the following numbers nearest'. Students were aware that the composition of this sentence was likely to make comprehension of the task difficult. So, over time, students noticed some of the pedagogical dilemmas that the authors of the textbooks faced; these aligned with the kind of decisions that Mason (2017) suggests teachers make when presenting examples to children.

Implications for mathematics teacher educators

Implications for the practice of mathematics teacher educators are considered under three headings which reflect the key features of the project design.

Study of Examples

The outcomes of this small-scale study suggest that the collective study of example sets is a valuable activity within a teacher education programme. Through the careful study of example sets there is potential for attention to shift away from the hunt for attractively presented worksheets to a critical examination of the particular examples presented.

A further implication for practice is that emphasis needs to be given to the professional decision making that is involved in choosing examples. One way to do this could be to provide tasks which are designed to initiate shared inquiry. Pairs of students could be asked to articulate a justification for their choice of an example set prior to teaching a lesson and then reflect on the outcomes of their choice of examples after the lesson. Or, in seminars students could be asked to justify the choice of one particular example set rather than another. A further development could be to ask students to make a systematic change to examples presented in textbooks or worksheets and reflect on the potential impact on children's learning.

Using textbooks as a source of examples

At the start of the research project, the textbook was simply a place to find sets of examples for students to consider. However, over time, it became clear that the study of examples presented in textbooks had the potential to provide effective professional development for beginner teachers and so could be central (rather than incidental) to the process of enhancing student choice of examples in lessons.

Using textbooks provides a single starting point for choosing a set of examples for a sequence of lessons, rather than individual puzzles, problems or questions on separate websites or worksheets. From that single starting point beginner teachers can consider amendments, additions and omissions from the example sets listed based on an evaluation of learning intentions for extended periods of time. There is also potential to shift away from a culture where textbook use is perceived as repetitive and tedious to one where textbook use is considered to have the potential to address the needs of learners through carefully considered, coherent learning sequences. Finally, there is also potential to promote a culture where the careful study of learner and teacher materials is seen as valuable professional development beyond an initial programme of instruction (Ma, 1999).

‘Noticing’ as an pedagogical strategy in mathematics teacher education

By consistently asking the question ‘what do you notice?’ the warrant of authority is transferred from the tutor to the students and validation of ideas is sought from within the group. During the research events students became increasingly aware of the complexity of the issues and engaged in sustained independent discussion of a range of ideas. Over time, the ‘discipline of noticing’ afforded the opportunity for students to identify and then examine in detail important aspects of mathematical examples, make links between the choice of examples and the context in which those examples are presented and to connect specific example sets with the broader principles of example choice and textbook design. The implication for practice is that by explicitly asking groups of students to self-validate, rather than seek validation from a tutor, there is potential for beginner teachers to take a more reflective approach to the discussion and critique of example sets.

Conclusion

It is unrealistic to expect fundamental growth in teacher knowledge after participation in a short professional learning activity, however evidence from each of the research- events support the conclusion that over time this group of students became ‘sensitised’ to notice more about examples presented in textbooks. The outcomes of this study lead to the recommendation that programmes of teacher education include explicit opportunities for beginner teachers to generate, explore and reflect upon examples for real or hypothetical classroom situations. The outcomes of this study suggest that such encounters provide rich and powerful learning experiences that lead to deeper awareness of aspects of creating and choosing examples for learners to consider. However, this study was limited to an exploration of new understanding in the context of the research group. In future studies, it would be valuable to explore how students used the knowledge gained to design their own example sets and look for patterns of change in students classroom practice, so that more general insight into the value of ‘noticing’ as an approach to teacher education can be identified.

References

Askew, M. and Wiliam, D. (1995) Recent research in mathematics education 5-16. London: Ofsted.

Askew, M. (2004) ‘Mediation and interpretation: Exploring the interpersonal and the intrapersonal in primary mathematics lessons’*, Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*. Bergen, Norway, 14-18 July*.* Bergen: Bergen University College. pp. 71-78.

Braun, V. and Clarke, V. (2006) ‘Using thematic analysis in psychology’, *Qualitative Research in Psychology*, 3(2), pp 77-101.

Cambridge Assessment (2016) *The Cambridge Approach to Textbooks* http://www.cambridgeassessment.org.uk/Images/cambridge-approach-to-textbooks.pdf. (Accessed: 4 August 2020).

Denzin, N. and Lincoln, Y. (2011) *Handbook of qualitative research.* California: Sage.

Doerr, H. (2000) ‘How can I find a pattern in this random data? The convergence of multiplicative and probabilistic reasoning’, *Journal of Mathematical Behaviour.* 18, pp. 431-454.

Fan, L. (2016) *Shanghai maths - The Shanghai maths project practice book year 4: For the English national curriculum*. London: Collins.

Gu, L. (1981) The visual effect and psychological implications of transformation of figures in geometry. *Annual conference of Shanghai Mathematics Association*. Shanghai, China.

Leung, A. (2003). 'Dynamic geometry and the theory of variation', in.Pateman, B., Dougherty, and J. Zilliox (eds.) *Proceedings of the 2003 joint meeting of PME and PMENA,3,* Honolulu: University of Hawaii. pp. 197 -204.

Ma, L. (1999) *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* Mahwah, NJ: Lawrence Erlbaum Associates.

Marton, F. and Booth, S. (1997*) Learning and awareness.* Mahwah, NJ: Lawrence Erlbaum Associates.

Mason, J. (2002) *Researching your own practice: The discipline of noticing.* Routledge: London.

Mason, J. (2017) 'Issues in variation theory and how it could inform pedagogical choices' in Huang, P. and Li, Y. (eds.) Teaching and Learning Mathematics through Variation. Rotterdam: Sense Publishers. pp.407-438.

NCETM (2015) Evaluation of the textbook project: Year one (January – July 2015)

Available at: http://www.mathshubs.org.uk/media/5202/ncp2-evaluation-final-sept-21.pdf (Accessed: 8 August 2020).

Polya, G. (1981) *Mathematics discovery: An understanding, learning, and teaching problem solving.* New York: John Willey & Son.

Savin- Baden, M. and Major, C. H. (2013) *Qualitative research: The essential guide to theory and practice.* London: Routledge.

Skemp, R. (1971) *The psychology of learning mathematics.* Harmondsworth: Penguin.

Simon, M. and Tzur, R. (2004) 'Explicating the role of mathematical tasks in conceptual learning: An elaborating of the hypothetical learning trajectory', *Mathematical Thinking and Learning,* 6, pp.91-104.

Stake, R.E. (2000) 'The case study method in social inquiry', in Gomm, R., Hammersley, M. and Foster, P. (eds.) *Case study methods*. London: Sage. pp.19-27.

Von Glasersfeld, E. (1995) *Radical constructivism: A way of knowing and learning.* London: Falmer.

Watson, A. and Mason, J. (2005) *Mathematics as a constructive activity: Learners generating examples.* Mahwah, NJ, USA: Erlbaum.

Watson, A. and Mason, J. (2006) 'Seeing an exercise as a single mathematical object: Using variation to structure sense making', *Mathematical Thinking and Learning,* 8(2), pp. 91-111.

Watson, A. and Thompson, D. (2015) 'Design issues related to text-based tasks', in. Watson, A. and Ohtani, M. (eds.) *Task design in mathematics education: An ICMI study 22.* Cham: Springer. pp.143-190.

Watson, A. (2017) 'Pedagogy of variations', in Huang, R. and Li, Y. (eds.) *Teaching and learning mathematics through variation.* Rotterdam: Sense Publishers. pp. 407-438.

Zodik, I. and Zaslavsky, O. (2008) 'Characteristics of teachers' choice of examples in and for the mathematics classroom', *Education Studies Mathematics*, 69(2), pp. 165–182.