# The Introduction of the Singapore Bar Model in Year 1 Problem Solving: A Personal Reflection

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**Abstract**

My study involved evaluating my influence on children's learning, through introducing the Singapore bar model to a group of Year 1 children and the impact this had on their problem solving. Since this is such a new area of study, I felt that the effectiveness of the pedagogical approaches undertaken, in order to make this accessible and beneficial, for Year 1 pupils, needed to be explored and the implications for future practice to be analysed.

Key findings: The use of the bar to model problems and number sentences allowed specific links between addition and subtraction to be made and some of the children were able to identify patterns and apply their understanding, engaging in higher order thinking and deepening understanding. However, pictorial representations needed to be supported with concrete experiences using manipulatives and scaffolded with questioning, in order for the children to access and benefit from the model.

## Introduction

Due to what has been described as the United Kingdom's "stagnation" (DfE, 2015) of performance in mathematics, compared to the rest of the world, the government has spent recent years analysing mathematics teaching in high performing jurisdictions, to discover what makes these systems effective and how this can be implemented into our own education system. The recommendations of the Vorderman report (Vorderman et al. 2011), which highlighted the extent to which this country is apparently trailing its economic competitors, with regard to mathematics education, led to the development of the new national curriculum (DfE, 2013), with a strong focus on mastery, rather than accelerated learning. The mastery approach is a model used in the high performing Asian systems, such as Shanghai and Singapore, with a key emphasis on slow, deep learning. One significant pedagogical approach, which has started to be imported into the education systems of the UK and other Western countries, as a direct result of international comparative studies, is the use of the Singapore Bar Model. Using models to represent word problems and number relationships has been a significant element of the mathematics curriculum in Singapore since the early 1980s (Clark, 2013). This visual approach to problem solving has been accredited as part of Singapore's success in global league tables and, as a result, the government are hoping that implementing the system into British classrooms will contribute towards close the gap in attainment. Although such bar modelling is first introduced to pupils, in Singapore, in Grade 3, where children are aged 8-9, research is currently being carried out in pilot schools across the UK, to introduce it in Year 1. During this study I aim to evaluate my influence on children's learning, through introducing the Singapore bar model to a group of Year 1 pupils, specifically regarding their approach to solving word problems.

## Literature review

*Government response to international data*

According to international league tables, such as PISA and TIMMS, the gap in attainment in mathematics of school children in Asian and Anglosphere countries has shown an overall increase

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over recent years (OECD, 2012; Mullis et al. 2012). Explaining and narrowing this gap has become an educational priority for the UK and other Western countries. Over the last decade, OFSTED have consistently identified pupils' lack of conceptual understanding in mathematics, across all key stages, and a reliance on factual recall of knowledge and procedural efficiency (DCSF, 2008; DfE, 2012). The aims of the current national curriculum (DfE, 2013) are set out to overcome this problem, with an emphasis on fluency, reasoning and problem solving. Children are expected to consolidate their understanding of the basics of mathematics, and develop their ability to apply their knowledge, before moving onto the next aspect, developing what Skemp (1989) refers to as relational understanding as opposed to instrumental understanding. This can be identified in the setting of objectives to specific year groups. Rather than accelerating onto the following year's objectives, pupils are now expected to deepen their understanding within the parameters of their own year group. This forms the fundamental basis for the mastery curriculum, followed in the Asian systems, where, in the early years of primary school, children become secure with number before progressing onto the next stage. (NCTL, 2014). Formal practice and drill are embedded within the culture of teaching, in order for children to master the basics and become fluent in recalling number facts.

In addition to the introduction of the revised curriculum, the government have also established a network of 32 maths hubs across the country, which have been funded to implement and lead other schools in developing an Asian style approach to mathematics teaching and learning, including using specialised teachers in each subject and following Singapore style text books to plan lessons, which provide a "coherent and structured programme." (DfE, 2015). Such consistency of using one variant model throughout schooling, like the bar model in problem solving, is fundamental to the design of the Singapore curriculum. The formation and purpose of the maths hubs is signified by the government view that there is "no reason why children in England cannot achieve the same standards in maths as those in Japan, Singapore and China," (DfE, 2014). The merits of the Asian systems are evident for the world to see when comparing academic attainment in world league tables.

In order for the UK to progress through the world rankings, the government action of importing these proven successful approaches could appear to be the logical way forward. However, Alexander (2014) questions the potential value of comparing global systems in league tables, such as PISA, since they do not consider the wider socio-cultural aspects that such systems are imbedded in. Countries are unique and distinct from one another. Therefore, what equates to a successful system in one country may not work as well in another. Eastern ideologies vary from those in the West and impact upon the way education is both valued and delivered. This is supported by the Vorderman report (2011, p29), which was central to the government education reforms in mathematics, advising that "international comparisons must be applied in the context of this country."

*Inconsistencies in adopting foreign approaches*

Despite the government drive towards a mastery design, curriculum expectations in the early stages of schooling vary significantly between UK and Asian systems. In Shanghai and Singapore, Grade 1 pupils, the equivalent age of English Year 2 pupils, are expected to count to 20. By the end of Year 1, English pupils are expected to count to and across 100. In Shanghai and Singapore, multiplication and division are not included until much later. However, in England, they now feature in Year 1 statutory requirements. Hanson (2014) argues that the Singapore approach cannot be successfully implemented within this curriculum for these reasons. She stresses that introducing the level of abstract maths to key stage 1 pupils, as set out in the curriculum, could potentially damage their cognitive development. The bar model is not introduced in Singapore until Grade 3, where children are aged 8-9. This logically coincides with the time at which, by Singapore standards, children are expected to have mastered the basics of number and are ready to develop new strategies and apply their understanding. If English children are expected to do more than their Asian counterparts in Year 1, then it could be considered appropriate to introduce the bar modelling strategy earlier. However, this may be considered contradictory to the mastery approach to learning. Although teachers are not able to progress children onto the curriculum objectives of the following year, the objectives themselves have been set in a way which means more content is being covered at an earlier stage. This becomes evident when comparing the current national curriculum to the previous one (DfES, 1999). As a result, it could be argued that acceleration has been government-imposed and, therefore, implementing Asian mathematics systems, without the solid foundation needed to build upon, will not necessarily yield the desired improvement in conceptual understanding and subsequent academic attainment for UK pupils.

*The significance of bar modelling*

The structure of the Singapore curriculum dictates the delayed introduction of the bar, specifically, to model problems. Its approach follows Piaget's (1974) constructivist theory that children construct their own knowledge, through experience – the assimilation and accommodation of ideas. The Curriculum is firmly based in representing mathematical concepts, as concrete, pictorial and abstract, according to children's developmental stage of thinking (Kheong, 2009) This links directly to Bruner's (1966) suggested phases of development: enactive, iconic and symbolic. Bar modelling fits into, what Bruner describes as, the iconic phase of development, where children begin to represent mathematical concepts in a more abstract way, through pictorial representations. In Singapore, children are deemed to be ready for this step towards abstract understanding by Grade 3. However, as stated previously, in England, contrasting curriculum demands have led to children being expected to internalise abstract mathematics at a younger age than their Singapore counterparts. In order to do so, introducing iconic representations, such as the bar model, may be beneficial at a younger age.

*Personal response to the literature*

Government trends towards the adoption of foreign policy could potentially conflict with the expectations of our own mathematics curriculum, in particular within Key Stage 1, where in Asia children are consolidating their understanding of number, while in the UK children are applying abstract thinking. However, adjusting practice to conform to imposed policy is not a new concept for education professionals and, therefore, finding ways to make it work for the children we teach should remain a part of our professional responsibility. As a result, this study aimed to investigate how I could introduce the Singapore bar model, to a group of Year 1 pupils, in a way which had a positive impact upon their learning. The bar model is just one aspect of the Singapore Mathematics curriculum and it is acknowledged that the methods adopted and context in which it took place, did not coincide with that of the Singapore curriculum. However, since this is such a new area of study, I felt that the effectiveness of the pedagogical approaches undertaken, in order to make this accessible and beneficial, for Year 1 pupils, needed to be explored and the implications for future practice to be analysed.

**Methodology**

In deciding upon an appropriate framework in which to conduct my research, I first considered the purpose of my research, that is to say, what I was hoping to achieve through carrying it out. This study involved introducing the Singapore bar model and evaluating its effect on children's learning, through a self-study, reflecting on my impact on the children's learning. The research undertaken focused on my own personal development and a commitment to improve my own practice, in order to result in the improved learning outcomes of both the children who took part in the study and also those I teach in the future. McNiff and Whitehead (2011 p.18) refer to action research as being a combination of "purposeful action with educational intent and testing the validity of any claims we make about the process." Unlike traditional research methods, for example experimental, where there is an end point or conclusion that can be drawn, through directly comparing sets of data, the purpose of my study was to reflect on the impact my actions and choices had on the children, at that point in space and time, and what to draw from this to take forward into future practice.

Hopkins (1993) suggests that the action research framework is most appropriate where a problem or scenario results in an intervention, the outcomes of which are then re-evaluated. This then proceeds onto another potential round of interventions. Had the research been longer, this would have formed the first cycle of action research, where my next cycle would have been informed through the reflection of this one, and so on. Lewin (1946, cited in Thomas, 2013) refers to a "circle of planning, action and fact finding resulting from the action," which in turn leads to social change. This cycle of action and reflection was central to the on-going teaching process over the course of the study. It is acknowledged that such reflections are personal and, as a result, will be biased, based on my own positionality within the system I was operating in.

In order to support and self-validate (McNiff and Whitehead, 2011) my reflections, the data collected involved observing, marking and evaluating. This triangulation (Thomas 2013) provided me with opportunities to reflect on different aspects of the children's learning and gave me a more accurate insight into their learning outcomes. Cohen et al. (2007) suggest that observation is powerful in gaining insight into situations. Being a participant observer allowed me to construct a depth of knowledge, regarding the progression of the children's understanding, through participating within their learning experience, which would not have been possible had I observed from outside the situation.

**Method**

I selected a focus group of seven Year 1 pupils. These were deemed, by the class teacher, to be high ability children in the class, based on a combination of their English and Maths abilities. I chose this group because the nature of the tasks I had prepared involved reading word problems and I felt that these children would be best suited, considering how young the children were and given the level of both maths and reading involved.

The children were given an initial problem solving activity sheet, Activity 1. This consisted of 10 questions with two of each type of problem: aggregation, augmentation, take away, counting back and find the difference. They had 15 minutes to answer the questions and had access to the following resources: number lines, hundred squares, counting bears, multilink cubes and Cuisenaire rods.

I was a participant observer and conducted a semi-structured observation, recording their talk during the activity, approach to the task, engagement and choice of resources on an observation form. My role as the observer was to keep the children on task, read questions if they were unsure what it said and use questioning to help guide them if they were stuck. I did not give them the answers or indicate if their answers were correct or not.

After this initial activity, my original plan was to teach this group of children to use the Singapore Bar Model in problem solving questions, for six 30 minutes sessions, after lunch, on consecutive school days. However, my reflection, following the second session, identified that the children were not engaging with the teaching. This could have been due, in part, to the time of day I was teaching, combined with the children's reluctance to participate in additional maths sessions, while the rest of the class enjoyed child-initiated activities. Therefore, instead, I taught the whole class for four 45 minute sessions during their normal maths timetable slot. During this teaching week, the TA and I worked with different ability groups in the class each day, as was consistent with normal classroom practice. The teaching involved introducing children to the concept of modelling number sentences and problems using a bar. The main focus was on aggregation, where the bar was representing the total of two groups that were combining. Representing take-away problems was also introduced to the top two groups, including the focus group. The sessions provided children opportunities to represent calculations using bars, through a range of manipulatives, as well as using iconic representations.

The focus group were finally given a problem solving sheet, Activity 2, in the same format to the initial activity. The same resources were available as before and they were given the same time limit. I recorded observations in the same format. My observing role was the same as it was previously.

**Limitations**

Because this was such a small study it would be inappropriate to make generalised conclusions based on these results. All of the children in the focus group are considered to be working at a high ability and are not representative of the class. Teaching problem solving over four lessons, prior to Activity 2, is likely to have impacted the result, despite the specific strategy being taught. Children's choices of strategies and resources may have been impacted by their desire to please me, as I had been teaching and observing them. The time period to conduct the research was very short and, therefore, it was impossible to introduce the bar model at the pace intended in the Singapore curriculum.

I fully considered and complied with the ethical guidelines set out by Canterbury Christ Church University, gaining full consent of the school and ascent of the pupils involved. I was authorised by the Head Teacher to use audio recording equipment during my observations.

**Results and analysis**

In order to begin to understand the effectiveness of my teaching, with regard to helping children progress in their understanding of modelling problems using a bar, first the data collected from Activities 1 and 2 was analysed. The comparison of children's scores from the activities undertaken pre and post teaching revealed surprising results. Figures 1a and b (below) show the overall increase in scores between Activity 1 and Activity 2.

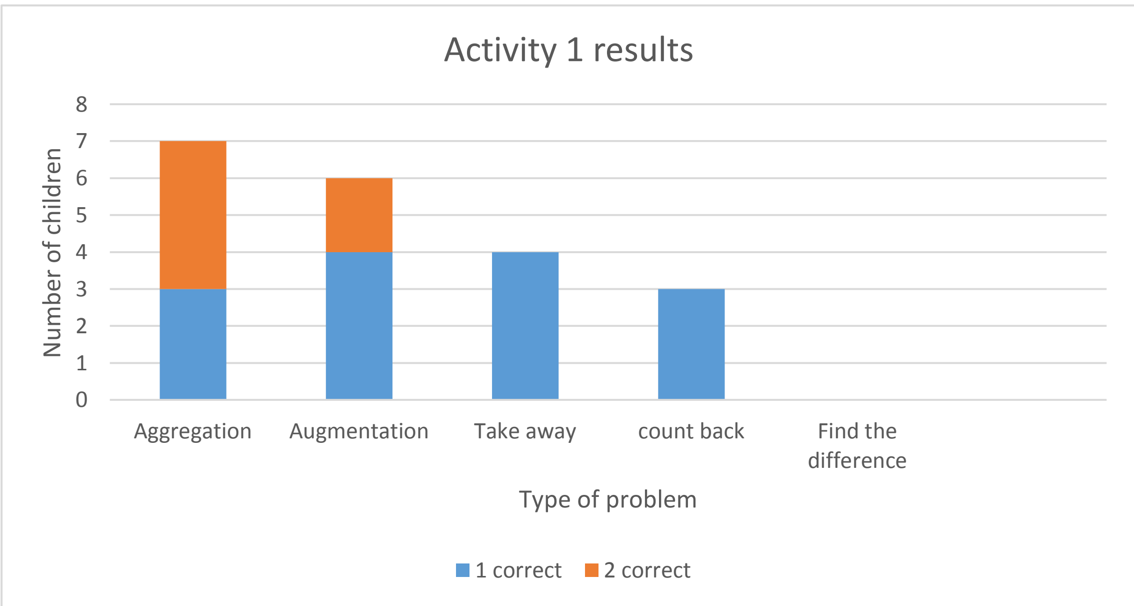


**Figure 1a.**

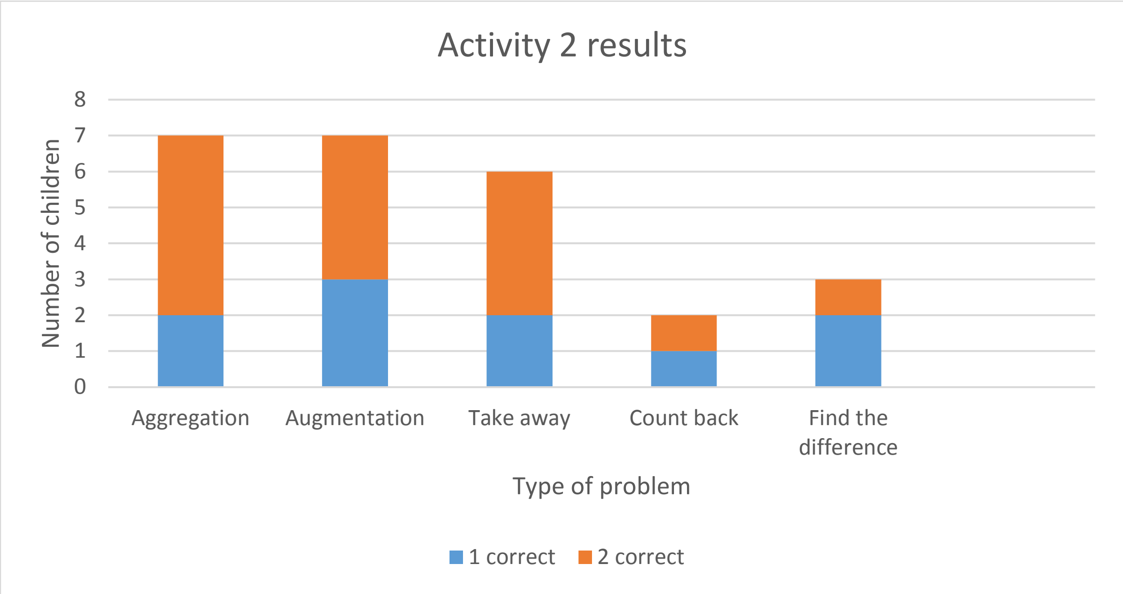


**Figure 1b.**

At first glance, it is apparent that more questions were answered correctly in Activity 2, particularly those relating to the models of subtraction. This was to be expected after having four lessons involving problem solving, where some of the teaching specifically centred on take away problems. However, closer analysis of the data reveals that only three of the children actually improved their scores between the two tests. These children scored significantly higher on Activity 2, while the other four children scored the same on both activities. Figures 2a and b (below) show the breakdown of each child's score on each of the activities.

**Table 1a.** Breakdown of Activity 1 results.

| Name | Aggregation | Augmentation | Take away | Count back | Find difference | Total |
|---|---|---|---|---|---|---|
| GG | 1 | 1 | 1 | 1 | 0 | 4 |
| CC | 1 | 1 | 1 | 1 | 0 | 4 |
| MJ | 2 | 0 | 0 | 0 | 0 | 2 |
| AM | 2 | 2 | 0 | 0 | 0 | 4 |
| AB | 2 | 2 | 1 | 0 | 0 | 5 |
| ER | 2 | 1 | 1 | 1 | 0 | 5 |
| TR | 1 | 1 | 0 | 0 | 0 | 2 |
| **Total** | **11** | **8** | **4** | **3** | **0** | **26** |

**Table 12b.** Breakdown of Activity 2 results.

| Name | Aggregation | Augmentation | Take away | Count back | Find difference | Total |
|---|---|---|---|---|---|---|
| GG | 2 | 1 | 1 | 0 | 0 | 4 |
| CC | 2 | 2 | 2 | 1 | 1 | 8 |
| MJ | 2 | 2 | 2 | 0 | 1 | 7 |
| AM | 2 | 1 | 1 | 0 | 0 | 4 |
| AB | 1 | 2 | 2 | 0 | 0 | 5 |
| ER | 2 | 2 | 2 | 2 | 2 | 10 |
| TR | 1 | 1 | 0 | 0 | 0 | 2 |
| **Total** | **12** | **11** | **10** | **3** | **4** | **40** |

The possible reasons for these differences will be examined below:

*Activity 1*

Activity 1 revealed that most of the children added the numbers in the question, regardless of the problem. During the activity, the children appeared unsure of what the questions were asking and were unable to approach them in a way which would lead them to the correct answer. They lacked strategies to unpick the questions. As a result, adding the numbers could have seemed like the easiest option for most of them, as it was an operation they were confident using and it resulted in them making a meaningful mark in the answer space. Although most of the children used resources during the activity, they selected these prior to reading the questions and did not change them during the activity. Therefore, their choices were not informed by which was best placed to help them solve the problem. Consequently, the resources served little purpose, other than to distract them. Nickson (2000) argues that although interacting with concrete manipulatives can promote thinking, the child does not necessarily gain a deeper conceptual understanding. Resources are not always appropriate to embody a concept and children may not have the guidance and scaffolding to use them in a way which supports their transition towards the abstract.

ER was able to successfully reason a counting back question and, with some scaffolding through questioning, modelled the problem with multilink cubes to support her reasoning. However, on the following question she proceeded without using cubes and got it wrong. This could suggest one of two things: that she lacked confidence in using the manipulatives available, without adult guidance; or she felt that using resources to help her make sense of the question would be a sign of weakness and lack of understanding.

*Activity 2*

CC, ER and MJ were able to demonstrate a much deeper level of understanding through their mathematical reasoning and application. In turn, this led to them all increasing their scores by 4 or 5 points. There are two significant factors to consider, regarding the increase for these children compared to the others, the first being the choice and manipulation of the resources available. The three high performing pupils all used cubes throughout the activity. In contrast to Activity 1, all three of them read the questions before selecting their resources and then proceeded to model the calculations in a similar way to how they had done so in class, by using two different coloured cubes to represent the two component parts of the bar. They were systematic in their approach to the problems and were able to adopt a strategy to help them both visualise and understand the question.

The second significant factor was their ability to apply this strategy to answer questions they were unfamiliar with. Previously, I discussed one of the limitations of this study as being that, through teaching problem solving all week, the children were more likely to be better at answering the questions, despite the method they used. However, the problems set during lessons focussed specifically on aggregation and take away. The children had received no prior teaching relating to find the difference. The scores from Activity 1 coincide with the children lacking the skills to decode these problems – none of them got either of the questions correct. Nevertheless, CC, MJ and ER were able to answer one or both of these questions correctly on Activity 2. Modelling comparison problems in 2 bars involves a different representation to the aggregation and take away problems involving one bar, which the children had been learning. Even so, the children were able to make connections with their existing learning and had the confidence to take risks in engaging with something they were unfamiliar with. Although they did not construct two separate bars and then compare the quantities, they were successful in adapting the strategy known to them for one problem and applying it to an unfamiliar problem. Through manipulating the cubes to make a bar, representing part of the problem, they were able to visualise and decode what the question was actually asking. This links to findings made through the Shanghai teacher exchange (NCTL, 2014), where a key feature of the Chinese children was their confidence and willingness to “attack” problems by using, what was described as, their “personal toolkit” of strategies.

GG also used multilink cubes and appeared to be attempting to model the problems in a bar. However, she was less successful and this could have been due to the lack of adult support to scaffold her reasoning. Although she knew the strategy she wanted to use, she struggled to link the concrete manipulation of the cubes to the abstract understanding of the problem. AB, AM and TR’s approaches were less systematic. They did not attempt to use the Singapore Bar Model, using concrete or pictorial representations. At the beginning of Activity 2, AM stated “I’m just going to add them all,” mirroring what many of the children had done during Activity 1. As with Activity 1, these children appeared to lack the skills to decipher what the questions were actually asking. They could not visualise the problems.

*Surprises*

AB’s lack of improvement between the 2 activities was unexpected, as he had shown evidence of making connections within his learning, in particular the relationship between adding and taking away, and applied his knowledge throughout the lessons. His surprisingly low score on Activity 2 could be attributed, in part, to his desire to finish first, whether or not his answers were correct. In addition, he was the only child who chose not to use any resources in either activity. On completing Activity 2, he commented “I didn’t even have to use anything.” This could be indicative of his attitude to resources as being things that those who struggle need to use. It may have been that he did want to appear inferior by using the resources available to him.

AB’s reluctance to use concrete resources could also be linked to his readiness to progress onto more iconic representations. This was also supported in his ability to use such representations during the taught sessions. Despite his ability to use iconic representations of the bar, during a teaching session, I introduced the multilink cubes to help him progress to an extension activity involving a take away problem. Through interacting with the concrete manipulatives, he was able to visualise the problem and move forward in his abstract understanding of what the question was asking, subsequently arriving at the correct answer. The choice of resource used to embody a concept and the point at which it is introduced are crucial to its effectiveness in progressing the children’s learning. At an earlier point in the learning, introducing manipulatives would have slowed his progress, as he was using the iconic representation of the bar effectively to solve aggregation problems. At the point at which this was no longer effective, in this instance, when making connections between aggregation and take-away, moving back from the iconic to the concrete enabled him to internalise the concept. Working independently in Activity 2, AB lacked the necessary teacher intervention to help scaffold his learning.

During the teaching sessions, the children used information from the word problems to label pre-drawn bars. The bars were pre-drawn because I felt it would take Year 1 children too much time to draw them themselves and that for some, a lack of fine motor skills could result in bars looking messy and causing confusion. However, in order to avoid influencing the children in their approach to solving the problems, I had not drawn any bars on either Activity 1 or 2. Because the children had not had the practice in constructing their own bars, some, particularly AB who preferred not to use resources, may not have associated using this method with these problems or may not have felt confident in constructing and labelling the bar without the template used during the lessons. None of the children used pictorial representations of the bar to help them solve the problems. Whereas ER, CC and MJ were able to use the multilink cubes to successfully model the problems in bars, some of the other children, like AB and GG, may have struggled linking the concrete to the abstract. Had such a template been drawn, it is possible that these children may have adopted this strategy more readily and scored much higher.

**My personal learning journey**

In this small scale study I have been able to reflect on my own practice and the impact of the pedagogical decisions I made in relation to introducing Year 1 children to the Singapore Bar Model. The use of the bar to model problems and number sentences allowed specific links between addition and subtraction to be made and some of the children were able to identify patterns and apply their understanding, engaging in higher order thinking and deepening understanding. It helped children to visualise problems and unpick questions. Some of the children gained confidence in attempting more difficult questions, since they developed strategies that they were able to apply. However, using iconic representations alone may not have been sufficient in supporting the children in their learning, especially when taking into account their level of cognitive development at age 5-6. It was through scaffolding the learning with the use of questioning, using manipulatives, and making key pedagogical decisions at specific moments of learning that the children were able to benefit from the model. The introduction of Asian textbooks could suggest that such decisions will be removed from the teacher in the future and simply following the instructions set out will lead to successful learning outcomes. However, I would hope that the government maintain their stance of supporting teacher autonomy (DfE, 2011). As a result, adjusting to the adoption of successful international strategies can be done in a way, where teachers respond to the learning needs of the children in their class and maximise the benefits for the children they teach, as in the case of any policy change, hence, maintaining professional integrity.

Through this study I have come to realise the importance of concrete experiences for children to gain conceptual understanding and the potential difficulties in bridging the gap between the concrete and abstract, particularly for young children. Children may be able to engage with model representations but they need to continue to revisit and consolidate with concrete experiences. Therefore, rather than viewing the bar model as the next step in progressing children's understanding from concrete to abstract, it could be viewed as being an important stage in a cycle of understanding, where children are guided to use manipulatives in a way to construct understanding, alongside the representation of a model, in order to bridge the intermittent state of development. The next steps, for this study, would be to carry out a longer piece of research, across a whole year, using my work here as a springboard into the next cycle of action research and could possibly form the basis of a future Masters paper. Comparing my impact and pedagogical choices in a Year 1 class and how this may look differently in a Year 6 class, could also be considered.

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